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Evolution of Dark Spatial Soliton in Quasi-phase-matched Quadratic Media*

WANG Fei-Yu, CHEN Xian-Feng,† CHEN Yu-Ping, YANG Yi, and XIA Yu-Xing
Department of Physics, the State Key Laboratory on Fiber-Optic Local Area Communication Networks and Advanced Optical Communication Systems, Shanghai Jiao Tong University, Shanghai 200240, China

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Abstract *We theoretically investigate the evolvement of dark spatial soliton with cascading quadratic nonlinearity in quasi-phase-matched second harmonic generation. It is shown that the dark solitary wave can propagate stably when background intensity is large enough, in which diffraction of beam can be balanced by the cascading quadratic nonlinearity. We also analyze the influence of phase-mismatch on the stability of dark soliton propagation.*

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The concept of optical solitons, which occur when dispersion (or diffraction) is balanced by nonlinearity, has now become ubiquitous in modern sciences. In nonlinear optics, the evolution of solitary waves with $\chi^{(3)}$ nonlinearity is described by nonlinear Schrödinger equation (NLSE). However, as has been already established, bright solitary waves can also exist with the cascaded second-order nonlinearities ($\chi^{(2)}:\chi^{(2)}$) in quadratic media. Some experimental and theoretical works have been done to demonstrate the existence of bright spatial soliton.[1−4] The soliton of this type has attracted growing attention owing to the possibility to generate large, intensity-dependent phase shifts, which lead to large effective $\chi^{(3)}$ nonlinearity. The experiments have proved that optical soliton due to cascading $\chi^{(2)}$ nonlinearity can obviously reduce the intensity threshold than soliton with $\chi^{(3)}$ nonlinearity.

In this paper, we first, to the best of our knowledge, prove that the dark spatial soliton exists when second-order nonlinearity with negative phase-mismatch is introduced.

In quadratic media, cascaded quadratic nonlinear process in second harmonic generation can be described as ($\omega-\omega\rightarrow 0$, $\omega+0\rightarrow\omega$ or $\omega+\omega\rightarrow 2\omega$, $2\omega-\omega\rightarrow\omega$). In this process, additive phase can be created, $\Delta\phi_{NL}=(\Gamma^2/\Delta k)z$ (Γ is the initial optical intensity of fundamental wave, $\Delta k=2k_1-k_2$, z is the propagation distance), just as that in third-order nonlinear process. So we can obtain the effective nonlinear index of refraction

$$n_{2\,\mathrm{eff}}=\frac{4\pi d_{\mathrm{eff}}^2}{c\varepsilon_0\lambda n_{2\omega}n_\omega^2\Delta k}\,,$$

which is $1\sim 2$ order lager than nonlinear index of refraction in third-order nonlinear process. Therefore we can see that when $\Delta k>0$, if a bright beam is launched into the media, the additive phase change resembles the intensity profile of the beam, forming an optical lens that increases the phase in the beam's center while leaving it unchanged in the beam's tails. This induced lens focuses the beam, so, a phenomenon like self-focus can exist and a bright spatial soliton may be achieved. Until now, most of research groups have focused on the study of bright spatial soliton. However, if phase-mismatch $\Delta k<0$, because intensity of the edge of the beam is larger than that of the center, the phase change in the center is still larger than that in the edge. So, the spatial dark soliton can be predicted.

Under the condition of slowly varying envelope approximation, we get the wave coupling equations of spatial soliton in a quasi-phase-matched (QPM) quadratic nonlinearity media. They used to be reduced in a normalized form,[5,6]

$$\mathrm{i}\frac{\partial a_1}{\partial\xi}+\frac{1}{2}\nabla_\perp^2 a_1+d(\xi)a_1^*a_2\exp(-\mathrm{i}\beta\xi)=0\,,$$

$$\mathrm{i}\frac{\partial a_2}{\partial\xi}+\frac{\alpha}{2}\nabla_\perp^2 a_2-\delta^\Lambda\delta\cdot\nabla_\perp^2 a_2+d(\xi)a_1^2\exp(\beta\xi)=0\,,\quad(1)$$

where a_1, a_2 are the normalized amplitudes of the fundamental and harmonic waves respectively, $\alpha=k_1/k_2$, k_1,k_2 are the wave numbers at the two frequencies. The parameter β ($\beta=k_1\eta^2\Delta k$) is proportional to the phase mismatch Δk ($\Delta k=2k_1-k_2+2\pi/\Lambda$, Λ is the one-order quasi-phase-matched grating period) and η is the characteristic beam transverse width. ξ is the propagation distance in the unit of $k_1\eta^2$. δ accounts for the Poynting vector walk-off when propagation is not along the crystal optical axes. We can set $\delta=0$ because Poynting vector walk-off is absent in typical QPM geometries. The function $d(\xi)$ stands for the effective nonlinear coefficients involved in QPM. $\nabla_\perp$ represents $\partial/\partial s$ in the situation of one dimension, here s

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†Correspondence author, E-mail: xfchen@sjtu.edu.cn

is the normalized transverse coordinate in the unit of η.

In our simulation, in order to investigate the evolution of dark soliton wave in QPM media, the input function of original fundamental wave is chosen as

$$f(r)=\begin{cases} \mathrm{e}^{-(r+200)^2}\,, & -300<r<-200\,,\\ 1\,, & -200<r<-2\pi\,,\\ \dfrac{1}{2}\Big(-\cos\Big(\dfrac{r}{2}\Big)+1\Big)\,, & -2\pi<r<2\pi\,,\\ 1\,, & 2\pi<r<200\,,\\ \mathrm{e}^{-(r-200)^2}\,, & 200<r<300a\,. \end{cases} \tag{2}$$

The input beam is shown in Fig. 1. The width of background is much wider than that of dark hollow, which is accord with the ideal situation of dark soliton. In the edge of input wave, we introduce a Gauss function in order to weaken the possible reflection in simulation. We introduce a π phase difference between two sides of the hollow, otherwise the hollow at the center will split into two ones during the evolvement.[7] In Fig. 1, the width of the hollow is about 8η. To investigate the beam evolvement, we integrate Eq. (1) numerically using a split-step approach. The linear part ($\nabla_{\perp}^2$) is integrated in the Fourier space and the nonlinear part is integrated by a fourth Runge–Kutta algorithm. We divide the propagation process into many steps. In every step, the diffraction effect is first considered exclusively, and then only nonlinear process is considered.

In our numerical simulation, we investigate the evolvement of spatial dark soliton with cascading χ^2 nonlinearity. We first investigate the evolvement of spatial dark soliton when the input intensity of original fundamental wave is relatively small. We set $\beta=-15$, while $a_1=1$, and for clear observation, we let the input beam propagate 10 diffraction lengths. The result of numerical simulation can be seen in Fig. 2(a). The input beam is expanded intensely and diffraction can be seen clearly. The result shows that when the intensity of the background is weak enough, the diffraction dominates and cannot be balanced by the cascading quadratic nonlinearity.

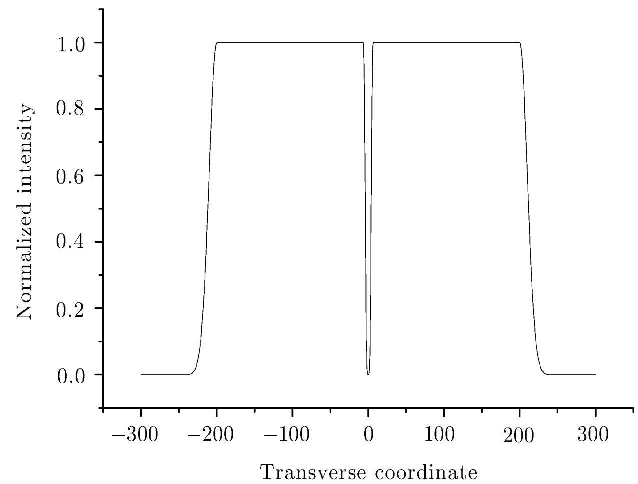

Fig. 1 The input intensity of the spatial dark soliton in the numerical simulation and there is a π phase difference between two sides of the hollow.

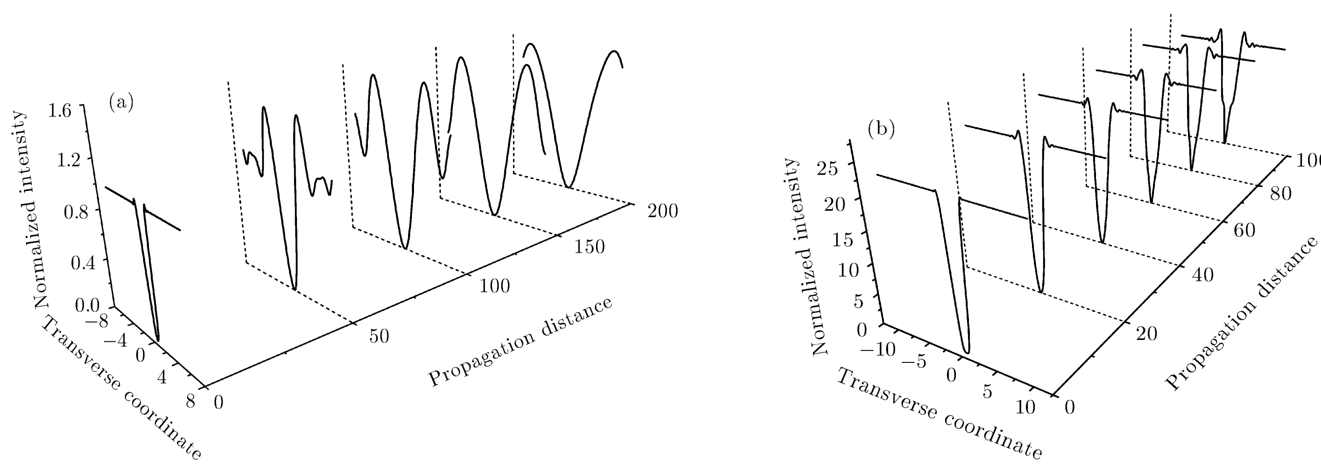

Fig. 2 (a) The evolvement of input pulse for 10 diffraction lengths. $a_1=1$ and we only draw the central part of the beam; (b) The evolvement of input pulse for 5 diffraction lengths. $a_1=5$ and we only draw the central part of the beam too.

When the intensity of fundamental beam increases, it becomes self-trapped owing to the increasing focusing effect by cascading quadratic nonlinearity. Figure 2(b) shows the evolvement of input beam in 5 diffraction lengths when normalized amplitude $a_1=5$ is adopted. It is clearly shown that in the propagation process the input pulse does not diffract and its original shape is maintained. In this case, the diffraction of the dark beam is almost balanced by cascading quadratic nonlinearity in QPM SHG. As we can see, the existing stable state in cascading quadratic nonlinearity is analogous to that of solitons (bright and dark) with $\chi^{(3)}$ nonlinearity and quadratic spatial bright solitons.[8−10] Furthermore, when we continue to increase the intensity of input beams, the evolvement becomes unstable and the beam splits into several parts. Detailed study of unstable self-trapped of dark beams in cascading quadratic nonlinearity is in process.

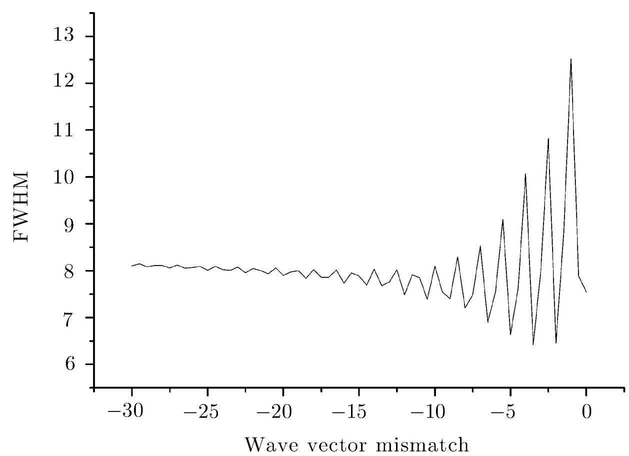

Fig. 3 The FWHM of fundamental wave at output section with different wave vector mismatch β. $a_1 = 5$ and the evolvement distance is 5 diffraction lengths.

Lastly, we analyze the influence of Δk ($\Delta k = 2k_1 - k_2 + 2\pi/\Lambda$) for spatial dark soliton propagation. Here we do not analyze Δk directly, instead we use β ($\beta = k_1\eta^2\Delta k$) and set $\eta = 0.02$ mm. In the theory of dark soliton with cascaded $\chi^{(2)}$ nonlinearity, wave vector match should be avoided. Here we set $a_1 = 5$ and let the pulse propagate 5 diffraction lengths. We only alter wave vector mismatch between fundamental wave and harmonic wave. We alter β from -30 to 0 and investigate the FWHM of output pulse. The result can be seen in Fig. 3. When Δk is small, namely β is small, as we can see, output pulse is very unstable and unregulated. In fact, in this situation dark soliton does not exist. And we can also see that when $|\beta|$ is larger than 15, the output pulse becomes stable and soliton is formed. The numerical simulation result is in accord with the theoretical predication.

In our numerical simulation, we design the model on the basis of periodically poled lithium niobate (PPLN). For example, the normalized intensity of fundamental wave corresponds to the magnitude of GW/cm^2, which is available by using pulsed solid lasers. For 5 diffraction lengths, the actual length is about 25 mm, which is the usual length for PPLN.

In summary, in our numerical simulation, we have investigated the evolvement of spatial dark soliton by cascaded second-order nonlinearity with negative wave vector mismatch. When the input intensity of fundamental wave is large enough, spatial dark solitons can propagate stably and do not diffract. On the contrary, spatial dark beams will diffract clearly. We also studied the influence of Δk for spatial dark solitons evolvement and concluded that a quantity of wave vector mismatch is needed to achieve a stable dark soliton.

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